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DIFFERENTIATION OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS.

BY WILLIAM T. JORDAN, WATERVILLE, MAINE.

To differentiate $$y = e^x. \tag{1}$$

$$y + dy = e^{x+dx} = e^x \,.\, e^{dx}. \tag{2}$$

Now $e^0 = 1$, and as dx is infinitesimal, e^{dx} will be but an infinitesimal greater than e^0, and as x is our variable, e^{dx} will be written $1 + dx$; $\therefore$

$$e^{dx} = 1 + dx. \tag{3}$$

Substituting this value of e^{dx} in (2) we have

$$y + dy = e^x(1 + dx) = e^x + e^x dx. \tag{4}$$

Subtracting (1) from (4), we have

$$dy = e^x\, dx. \tag{5}$$

To differentiate $y = a^x$ with reference to the Napierian system, we have

$$a = e^{\log_e a}. \tag{6}$$

Substituting $e^{\log_e a}$ for a from (6), we have

$$y = e^{\log_e ax}. \tag{7}$$

Differentiating as above we have

$$dy = e^{\log_e ax} \log_e a dx; \tag{8}$$

but $a = e^{\log_e a}$, $\therefore$

$$dy = a^x \log_e a dx. \tag{9}$$

To differentiate $y = a^x$ (10) with regard to any system. We have, if z be the base of the system, $a = e^{(\log_z a) \div m_z}$. Substituting this value of a in (10), we have

$$y = e^{(\log_z ax) \div m_z}. \tag{11}$$

Defferentiating as before, we have

$$dy = e^{(\log_z ax) \div m_z}(\log_z a dx) \div m_z. \tag{12}$$

Now $a = e^{(\log_z a) \div m_z}$; $\therefore$

$$dy = \frac{a^x \log_z a dx}{m_z}. \tag{13}$$

To differentiate the logarithm of a variable, let $y = e^x$, therefore

$$x = \log_e y;\ \therefore dx = d(\log_e y). \tag{14}$$

Now $dy = e^x dx$, $\therefore$ from (14) $dy = y dx$; $\therefore dx = dy \div y$, but $dx = d(\log_e y)$,

$$\therefore d(\log_e y) = \frac{dy}{y}. \tag{15}$$

Let z be the base of any aystem, then $\log_z y = m_z(\log_e y)$, $\therefore d(\log_z y) = m_z \times (\log_e y)$, $\therefore d(\log_z y) = m_z\,(dy \div y)$, the differential of a logarithm in any system.